BROJNI REDOVI

106. Ispitati konvergenciju redova

a) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^3+1}}{n}$

→ opšti član je $a_n = \frac{\sqrt[3]{n^3+1}}{n}$

JORDI LABANDA

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt[3]{n^3(1+\frac{1}{n^3})}}{n} = \lim_{n\to\infty} \frac{n\cdot\sqrt[3]{1+\frac{1}{n^3}}}{n} = 1 \neq 0$$

→ red $\sum_{n=1}^{\infty} a_n$ ne konvergira, tj divergira

→ Da bi red konvergirao → njegov opsti član mora da teži nuli

b) $\sum_{n=1}^{\infty} \frac{n+1}{2n+3}$

→ opsti član $a_n = \frac{n+1}{2n+3}$

$\lim_{n\to\infty} \frac{n+1}{2n+3} = \frac{1}{2} \neq 0 \Rightarrow$ red $\sum_{n=1}^{\infty} \frac{n+1}{2n+3}$ divergira

c) $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$ → niz parcijalnih suma ne konvergira → ne konvergira ni brojni red

$a_n = \ln \frac{n}{n+1} = \ln n - \ln(n+1)$

$S_n = a_1 + a_2 + \dots + a_n = \ln 1 - \cancel{\ln 2} + \cancel{\ln 2} - \cancel{\ln 3} + \dots + \cancel{\ln n} - \ln$

$= \underset{0}{\ln 1} - \ln(n+1) = -\ln(n+1)$

$\lim_{n\to\infty} S_n = -\lim_{n\to\infty} \ln(n+1) = -\infty$ →

Niz $(S_n)_{n\in\mathbb{N}}$ ne konvergira, pa ne konvergira ni red $\sum_{n=1}^{\infty} a_n$

d) $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$ $\qquad a_n = \ln \frac{n^2+1}{2n^2+1}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln\left(\frac{n^2\left(1+\frac{1}{n^2}\right)}{n^2\left(2+\frac{1}{n^2}\right)}\right) = \ln\frac{1}{2} \neq 0$$

$\Rightarrow$ red $\sum_{n=1}^{\infty} a_n$ ne konvergira

(107.) Naći sumu reda $\sum_{n=1}^{\infty} a_n$, gdje je ~~a_n~~

$$a_n = \sqrt{n+2} - 2\sqrt{n+1} + \sqrt{n}$$

$\Rightarrow$ posmatra se niz parcijalnih suma

$$S_n = a_1 + a_2 + \dots + a_n$$

$$S_n = \left(\sqrt{3} - 2\sqrt{2} + \sqrt{1}\right) + \left(\sqrt{4} - 2\sqrt{3} + \sqrt{2}\right) + \left(\sqrt{5} - 2\sqrt{4} + \sqrt{3}\right) + \dots$$

$$+ \left(\sqrt{n} - 2\sqrt{n-1} + \sqrt{n-2}\right) + \left(\sqrt{n+1} - 2\sqrt{n} + \sqrt{n-2}\right) +$$

$$+ \sqrt{n+2} - 2\sqrt{n+1} + \sqrt{n}$$

$$S_n = 1 - \sqrt{2} - \sqrt{n+1} + \sqrt{n+2}$$

gr. vr. niza parc. suma

racionališemo

$$\Rightarrow \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \sqrt{2} + \left(\left(\sqrt{n+2} - \sqrt{n+1}\right) \cdot \frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+2}+\sqrt{n+1}}\right)\right) =$$

$$= \lim_{n\to\infty}\left(1 - \frac{1}{\sqrt{n+2}+\sqrt{n+1}}\right) = 1 - \sqrt{2}$$

$\Rightarrow$ Niz S_n konvergira pa konvergira i red $\sum_{n=1}^{\infty} a_n$

(108) Naći sumu reda $\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)}$

$$a_n = \frac{1}{(3n-2)(3n+1)} = \frac{A}{3n-2} + \frac{B}{3n+1}$$

$$A = \frac{1}{3} \; ; \quad B = -\frac{1}{3}$$

$$a_n = \frac{1}{3}\left(\frac{1}{3n-2} - \frac{1}{3n+1}\right)$$

$$\rightarrow S_n = a_1 + a_2 + \dots + a_n =$$

$$= \frac{1}{3}\left(\frac{1}{1} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{7}} + \dots + \cancel{\frac{1}{3n-2}} - \frac{1}{3n+1}\right) =$$

$$= \frac{1}{3}\left(1 - \frac{1}{3n+1}\right)$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{3}\left(1 - \frac{1}{3n+1}\right) = \frac{1}{3}$$

$\rightarrow$ Niz parcijalnih suma (S_n) konvergira $\rightarrow$

$\rightarrow$ konvergira i red $\sum_{n=1}^{\infty} a_n$

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(109) Naći sumu reda $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{2^{n-1}} = \sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$$

$$\rightarrow a_n = \left(-\frac{1}{2}\right)^{n-1} \; ; \quad S_n = a_1 + a_2 + \dots + a_n$$

$$S_n = 1 - \frac{1}{2} + \frac{1}{4} + \dots + \left(-\frac{1}{2}\right)^{n-1} = 1 \cdot \frac{1 - \left(-\frac{1}{2}\right)^n}{1 - \left(-\frac{1}{2}\right)} =$$

$$= \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^n\right)$$

$$\lim_{n\to\infty} \frac{2}{3}\left(1 - \underset{0}{\left(-\frac{1}{2}\right)^n}\right) = \frac{2}{3} \rightarrow \text{niz } S_n \text{ konvergira}$$

pa konvergira i red $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{2^{n-1}}$

(110) Ispitati konvergenciju i naći sumu reda $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

$a_n = \frac{e^n}{3^{n-1}} = e\left(\frac{e}{3}\right)^{n-1}$

→ Ovo je geometrijski red → $e, e\frac{e}{3}, e\left(\frac{e}{3}\right)^2, \dots$

→ $q = \frac{e}{3}$ → $|q| < 1$ → Dati geometrijski red konvergira.

(111) Ispitati konvergenciju reda $\sum_{n=1}^{\infty}\left(\frac{1}{e^n}+\frac{1}{n(n+1)}\right) =$

$= \sum_{n=1}^{\infty} \frac{1}{e^n} + \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

→ opšti član $a_n = \frac{1}{e^n} + \frac{1}{n(n+1)}$

→ Neka je $b_n = \frac{1}{e^n}$ i $c_n = \frac{1}{n(n+1)}$

$\sum_{n=1}^{\infty} \frac{1}{e^n}$ konvergira kao geometrijski red kod kojeg je $q = \frac{1}{e} < 1$

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

$c_n = -\frac{1}{n+1} + \frac{1}{n} \rightarrow \frac{1}{n} - \frac{1}{n+1}$

$S_n = c_1 + c_2 + \dots + c_n = 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \dots + \cancel{\frac{1}{n}} - \frac{1}{n+1} =$

$= 1 - \frac{1}{n+1}$

$\lim_{n\to\infty} S_n \left(1 - \cancel{\frac{1}{n+1}}\right) = 1$ → niz S_n konv →

→ konv. i red $\sum_{n=1}^{\infty} c_n$

$$\left.\begin{array}{l}\sum_{n=1}^{\infty} b_n \text{ konv} \\ \sum_{n=1}^{\infty} c_n \text{ konv}\end{array}\right\} \Rightarrow \text{konv. i red } \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (b_n + c_n)$$

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(112) Ispitati konvergenciju reda $\sum_{n=1}^{\infty}\left(\frac{3}{5^n}+\frac{2}{n}\right)$

Hiperharmonijski red $\sum_{n=1}^{\infty}\frac{1}{n^p}$ konv. za $p > 1$, div za $p \leq 1$

$a_n = \frac{3}{5^n} + \frac{2}{n}$, $b_n = \frac{3}{5^n}$, $c_n = \frac{2}{n}$

$\sum_{n=1}^{\infty} b_n = 3 \sum_{n=1}^{\infty} \frac{1}{5^n}$; → red $\sum_{n=1}^{\infty}\frac{1}{5^n}$ konvergira kao geometrijski red za koji je $q = \frac{1}{5}$ i $|q| < 1$ → pa konvergira i red $3 \cdot \sum_{n=1}^{\infty}\frac{1}{5^n}$, tj. red $\sum_{n=1}^{\infty} b_n$

$\sum_{n=1}^{\infty}\frac{1}{n}$ divergira kao hiperharmonijski za koji je $p=1$ → odavde slijedi da divergira i red $\sum_{n=1}^{\infty}\frac{1}{n}$, tj. red $\sum_{n=1}^{\infty} a_n$.

$$\left.\begin{array}{l}\text{Red } \sum_{n=1}^{\infty} b_n \text{ konv.} \\ \text{Red } \sum_{n=1}^{\infty} c_n \text{ diverg}\end{array}\right\} \rightarrow \text{red } \sum_{n=1}^{\infty}\left(\frac{3}{5^n}+\frac{2}{n}\right)$$

divergira

Redovi sa pozitivnim članovima

(113) Ispitati konvergenciju reda:

a) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

$\rightarrow a_n = \frac{1}{2n-1}$; Neka je $b_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{2n-1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2} > 0$

$\rightarrow$ Redovi suma $\sum_{n=1}^{\infty} b_n$ i $\sum_{n=1}^{\infty} a_n$ su ekvikonvergentni

Red $\sum_{n=1}^{\infty} \frac{1}{n}$ divergira kao hiperharmonijski red

za koji je $p=1$ $\rightarrow$ pa divergira i red $\sum_{n=1}^{\infty} a_n$

---

b) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$

$a_n = \frac{1}{n\sqrt{n+1}}$; Neka je $b_n = \frac{1}{n\sqrt{n}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n\sqrt{n+1}}}{n\sqrt{n}} = 1 > 0 \rightarrow$ Redovi

$\sum_{n=1}^{\infty} a_n$ i b_n su ekvikonvergentni

$\rightarrow$ Red $\sum_{n=1}^{\infty} b_n$ je konvergentan kao hiperharmonijski za koji je $p = \frac{3}{2} > 1$, pa konvergira i red $\sum_{n=1}^{\infty} a_n$

(114) Ispitati konvergenciju reda $\sum_{n=2}^{\infty} \frac{\sqrt{n+2}-\sqrt{n}}{n^{\alpha}}$

$a_n = \frac{\sqrt{n+2}-\sqrt{n}}{n^{\alpha}} \longrightarrow$ Racionališemo

$a_n = \frac{n+2-n}{n^{\alpha}(\sqrt{n+2}+\sqrt{n})} = \frac{2}{n^{\alpha}(\sqrt{n+2}+\sqrt{n})}$

$\longrightarrow$ neka je $b_n = \frac{2}{n^{\alpha}\sqrt{n}}$

$\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \frac{\frac{\cancel{2}}{\cancel{n^{\alpha}}(\sqrt{n+2}+\sqrt{n})}}{\frac{\cancel{2}}{\cancel{n^{\alpha}}\sqrt{n}}} = \lim\limits_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+2}+\sqrt{n}} =$

$= \lim\limits_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}\left(\sqrt{1+\frac{2}{n}}+1\right)} = \frac{1}{2}$

$0 < \lim\limits_{n\to\infty} \frac{a_n}{b_n} < +\infty \longrightarrow \sum_{n=2}^{\infty} a_n$ i $\sum_{n=2}^{\infty} b_n$ su

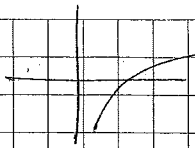
ekvikonvergentni

$\longrightarrow$ Red $\sum_{n=2}^{\infty} \frac{1}{n^{\alpha+1/2}}$ konvergira za $\alpha+\frac{1}{2}>1$

$\longrightarrow$ Red $\sum_{n=2}^{\infty} b_n$ konv. za $\alpha>\frac{1}{2}$

$\longrightarrow$ $\sum_{n=2}^{\infty} a_n$ konvergira za $\alpha>\frac{1}{2}$

(115.) $\sum_{n=2}^{\infty} \frac{\ln}{n}$



$\ln x > 1, \forall x > e$

$\forall n > 3$

$a_n = \frac{\ln n}{n}$

$\frac{1}{n} < \frac{\ln n}{n}, \forall n \geq 3$ ⟶ Kriterijum uporedivanja

$b_n = \frac{1}{n}$

$b_n < a_n \; \forall n \geq 3$

⟶ Red $\sum_{n=2}^{\infty} b_n$ divergira kao hiperharmonijski za koji je $p = 1$

1° $b_n < a_n, \forall n \geq 3$

2° $\sum_{n=2}^{\infty} b_n$ divergira

$\Longrightarrow$ po poredbenom kriterijumu $\sum_{n=2}^{\infty} a_n$ divergira

(116.) $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n}$ ⟶ $\frac{1}{n \cdot 2^n} \leq \frac{1}{2^n}, \forall n \in \mathbb{N}$

$a_n = \frac{1}{n \cdot 2^n}$ $b_n = \frac{1}{2^n}$ ⟶ Kriterijum uporedivanja

Red $\sum_{n=1}^{\infty} b_n$ je geom. red za koji je $q = \frac{1}{2}$, a kako je $|q| < 1$ ⟶ to ovaj red konvergira

1° $a_n \leq b_n, \forall n \in \mathbb{N}$

2° $\sum_{n=1}^{\infty} b_n$ konv

$\Longrightarrow$ po kriterijumu uporedivanja ovaj red ($\sum_{n=1}^{\infty} a_n$ konv.)

117. $\sum_{n=1}^{\infty} \frac{1}{3^n+n}$ → $\frac{1}{3^n+n} \le \frac{1}{3^n}$, $\forall n \in \mathbb{N}$

$a_n = \frac{1}{3^n+n}$

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118. $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2n}$ → $a_n = \frac{\sin 2n}{1+2^n} < \frac{1}{1+2^n} < \frac{1}{2^n}$

→ Kriterijum uporedivanja

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119. $\sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}}$ → Dalamberov kriterijum

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \begin{cases} < 1 \to \text{konvergira} \\ > 1 \to \text{divergira} \\ = 1 \to \text{neki drugi krit.} \end{cases}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{((n+1)!)^2}{2^{(n+1)^2}}}{\frac{(n!)^2}{2^{n^2}}} =$$

$$= \lim_{n\to\infty} \frac{\frac{(n+1)^2 \cdot \cancel{(n!)}^2}{\cancel{2^{n^2}} \cdot 2^{n+1}}}{\frac{\cancel{(n!)}^2}{\cancel{2^{n^2}}}} = \lim_{n\to\infty} \frac{(n+1)^2}{2^{n+1}} = 0 < 1$$

→ brže raste exp. f-ja → ψ

Red suma $\sum_{n=1}^{\infty} a_n$ konvergira po D. k.

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120. $\frac{4}{2} + \frac{4\cdot 7}{2\cdot 6} + \frac{4\cdot 7\cdot 10}{2\cdot 6\cdot 10} + \cdots$ → Dalamberov kriterijum

→ $\sum_{n=1}^{\infty} \frac{4\cdot 7 \cdots (3n+1)}{2\cdot 6 \cdots (4n-2)}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\frac{4\cdot 7 \cdots \cancel{(3n+1)}(3n+4)}{2\cdot 6 \cdots \cancel{(4n-2)}(4n+2)}}{\frac{4\cdot 7 \cdots \cancel{(3n+1)}}{2\cdot 6 \cdots \cancel{(4n-2)}}} = ~~3n~~$$

$$= \lim_{n\to\infty} \frac{3n+4}{4n+2} = \frac{3}{4} < 1 \xrightarrow[D.K]{} \sum_{n=1}^{\infty} a_n \text{ konv.}$$

(121.) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$, $a_n = \frac{n!}{n^n}$ → Dalamberov kriterijum

$$\lim_{n\to\infty} \frac{a_{(n+1)}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} = \lim_{n\to\infty} \frac{n^n}{(n+1)^n} =$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n} =$$

$$= \lim_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^n\right)^{-1} = e^{-1} = \frac{1}{e} < 1 \rightarrow$$

→ po D. k → $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ konv.

(122.) $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n(n-1)}$ → Košijev kriterijum

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \left(\frac{n-1}{n+1}\right)^{n-1} = \lim_{n\to\infty} \left(1+\frac{n-1}{n+1}-1\right)^{n-1} =$$

$$= \lim_{n\to\infty} \left(1+\frac{-2}{n+1}\right)^{\frac{n+1}{-2}\cdot\frac{-2}{n+1}(n-1)} =$$

$$= e^{\lim_{n\to\infty} \frac{-2n+2}{n+1}} = e^{-2} = \frac{1}{e^2} < 1$$

→ Po Koš krit $\sum_{n=1}^{\infty} a_n$ konv.

(123.) $\sum_{n=1}^{\infty} \left(\frac{1+\cos n}{2+\cos n}\right)^{2n-\ln n}$ → Košijev kriterijum

$$\frac{1+\cos n}{2+\cos n} = 1 - \frac{1}{2+\cos n} \leq 1 - \frac{1}{2+1} = 1-\frac{1}{3} = \frac{2}{3}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} \leq \lim_{n\to\infty} \sqrt[n]{\left(\frac{2}{3}\right)^{2n-\ln n}} = \rightarrow$$

→ more Lopit. pr

$$= \lim_{n\to\infty} \left(\frac{2}{3}\right)^{2-\overset{0}{\boxed{\frac{\ln n}{n}}}} = \frac{4}{9} < 1$$ → na osnovu

Koš. krit. red a_n konv.

(124.) $\sum_{n=1}^{\infty} \left(\frac{n-1}{n}\right)^n$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{n-1}{n} = 1$$ → ~~Koš~~ijev krit

$$a_n = \left(\frac{n-1}{n}\right)^n$$

$$\to \lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{n-1}{n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{-1}{n}\right)^{-n(-1)} =$$

$$= e^{-1} = \frac{1}{e} \neq 0$$

$$\to \lim_{n\to\infty} a_n \neq 0 \to \sum_{n=1}^{\infty} a_n$$ ne konvergira

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(125.) $\sum_{n=1}^{\infty} \frac{n!}{(a+1)(a+2)\cdots(a+n)}$; $a>0$

Dalamberov krit

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)\cancel{n!}}{\cancel{(a+1)(a+2)\cdots(a+n)}(a+n+1)}}{\frac{\cancel{n!}}{\cancel{(a+1)(a+2)\cdots(a+n)}}} =$$

$$= \lim_{n\to\infty}$$

→ 125, 126, 127, 128, 129, 130, 131, 132, 133, 134

~~posle funkcija~~!

sl. str.

$$\boxed{125.} \quad \sum_{n=1}^{\infty} \frac{n!}{(a+1)(a+2)\cdots(a+n)} \quad ; \quad a>0$$

Dalamb.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\dfrac{(n+1)\cdot\cancel{n!}}{\cancel{(a+1)(a+2)\cdots(a+n)}(a+n+1)}}{\dfrac{\cancel{n!}}{\cancel{(a+1)(a+2)\cdots(a+n)}}} =$$

$$= \lim_{n\to\infty} \frac{n+1}{a+n+1} = 1 \Rightarrow$$ Dalamberov kriterijum ne daje odgovor.

→ Primijenimo Rabov test ↗ ako je $\lim > 1$ kon
$\lim < 1$ diverg.

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}} - 1\right) = \lim_{n\to\infty} n\cdot\left(\frac{a+n+1}{n+1} - 1\right) =$$

$$= \lim_{n\to\infty} n\cdot\frac{a}{n+1} = \lim_{n\to\infty} a\cdot\frac{n}{n+1} = a$$

Za $a > 1$ → konvergira

Za $0 < a < 1$ red divergira

ako je $a = 1$ – dobijamo brojni red

$$\sum_{n=1}^{\infty} \frac{n!}{2\cdot 3\cdots(1+n)} \rightarrow a_n = \frac{1}{n+1}$$

$$b_n = \frac{1}{n}, \quad \lim_{n\to\infty} \frac{a_n}{b_n} = \lim \frac{n}{n+1} = 1 \Rightarrow$$

$\Rightarrow \sum a_n$ i $\sum b_n$ ekvikonvergentni

Red $\sum b_n$ div. kao hiperharm.

$\Rightarrow$ i $\sum a_n$ diverg.

126

$$\sum_{n=1}^{\infty} \frac{(-2)^n + 3n^2}{3^n}$$

$$\underbrace{\sum_{n=1}^{\infty} -\left(\frac{2}{3}\right)^n}_{a_n} + \underbrace{\sum_{n=1}^{\infty} \frac{3n^2}{3^n}}_{b_n}$$

$\sum a_n$ je geom. red $|q| = \left|-\frac{2}{3}\right| < 1$

$\Rightarrow a_n$ konvergira 1°

$b_n = \frac{3n^2}{3^n}$ — Dalamberov krit

$\lim\limits_{n\to\infty} \frac{b_{n+1}}{b_n} = \frac{1}{3} < 1 \Rightarrow$ konvergira 2°

iz 1° i 2° $\Rightarrow$ red $\sum(a_n + b_n)$ konvergira

127.

$$\sum_{n=1}^{\infty} n \cdot e^{-n^2}$$

Košijev integralni kriterijum

$a_n = n \cdot e^{-n^2}$

Neka je $f(x) = x \cdot e^{-x^2}$. Tada je $a_n = f(n), \forall n \in N$

1° $f(x) = \frac{x}{e^{x^2}}$; f je neprekidna kao kompozicija elementarnih u oblasti definisanosti, pa i na $[1, +\infty)$

2° $f(x) = \frac{x}{e^{x^2}} > 0, \ \forall x \in [1, +\infty)$

$x_1 < x_2$; $f(x_1) \geq f(x_2)$

3° $f'(x) = \frac{1-2x^2}{e^{x^2}} < 0 \Leftrightarrow 1 - 2x^2 < 0$

$\Leftrightarrow x \in \left(-\infty, -\frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2}, +\infty\right)$

sl. str. → 2

Odavde slijedi da je $f'(x) < 0$ i za $\forall x \in [1, +\infty)$ pa je fja f na $[1, +\infty)$ opadajuća.

iz 1°, 2° i 3° na osnovu Košijevog integralnog kriterijuma slijedi da red $\sum_{n=1}^{\infty} a_n$ konvergira akko $\int_1^{+\infty} f(x)\,dx$ konvergira.

$$\int_1^{+\infty} f(x)\,dx = \lim_{B\to+\infty} \int_1^{B} x\cdot e^{-x^2}\,dx = \left| \begin{array}{l} \text{smjena } x^2 = t \\ 2x\,dx = dt \end{array} \right| = \dots =$$

$$= \lim_{B\to+\infty} \left(-\frac{1}{2}\cdot \underset{0}{\cancel{\frac{1}{e^{B^2}}}} + \frac{1}{2}\cdot\frac{1}{e} \right) = \lim_{n\to\infty} \frac{1}{2e} = \frac{1}{2e}$$

$\Rightarrow$ integral $\int_1^{+\infty} f(x)\,dx$ konv, pa i red $\sum a_n$ konv.

(128) $\sum_{n=2}^{\infty} \frac{1}{n\cdot\ln^2 n} \to a_n = \frac{1}{n^2\cdot\ln^2 n}$

Neka je $f(x) = \frac{1}{x\cdot\ln^2 x}$.

$f(n) = a_n,\ \forall n \geq 2$

1° neprekidna

$f(x) = \frac{1}{x\cdot\ln^2 x} \Rightarrow$ neprekidna na int. $[2, +\infty)$

2° znak

$f(x) = \frac{1}{\underset{>0}{x}\cdot\underset{>0}{\ln^2 x}} > 0,\ \forall x \in [2, +\infty)$

3° $f'(x) = -\frac{\ln x + 2}{x^2\cdot\ln^3 x} < 0,\ \forall x \in [2, +\infty)$

$f \downarrow$ na $[2, +\infty)$

Iz 1°, 2° i 3° $\Rightarrow$ red $\sum\limits_{n=2}^{\infty} a_n$ konv. akko $\int\limits_{2}^{+\infty} f(x)\,dx$ konv.

$$\int\limits_{2}^{+\infty} \frac{1}{x\cdot \ln^2 x}\,dx = \lim_{B\to+\infty} \int\limits_{2}^{B} \frac{1}{x\cdot \ln^2 x}\,dx = \left\lceil \ln x = t \right\rfloor = \dots =$$

$$= \lim_{B\to+\infty}\left(-\frac{1}{\ln B} + \frac{1}{\ln 2}\right) = \frac{1}{\ln 2} \Rightarrow$$

$\Rightarrow$ ovaj integral konvergira $\Rightarrow$ red $\sum\limits_{n=2}^{\infty} a_n$ konv.

znak se mijenja ← **Alternativni redovi**

(128)

$$1 - \frac{3}{2} + \frac{5}{4} - \frac{7}{8}$$

* Samo Lajbnicov kriterijum

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{2n-1}{2^{n-1}}$$

1° posmatramo niz $b_n = |a_n| = \dfrac{2n-1}{2^{n-1}}$

$$b_n - b_{n+1} = \frac{2n-1}{2^{n-1}} - \frac{2n+1}{2^n} = \frac{2(2n-1)-2n-1}{2^n} = \frac{2n-3}{2^n} > 0,\quad \forall n \geq 2$$

→ (izbacujemo b_1, odnosno a_1, ali to ne utiče na konvergenciju.)

$b_n > b_{n+1}$, $\forall n \geq 2 \Rightarrow$ Niz (b_n) je opadajući

2° $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \dfrac{2n-1}{2^{n-1}} = 0$;

Iz 1° i 2° na osnovu Lajbnicovog kriterijuma slijedi da red $\sum\limits_{n=1}^{\infty} a_n$ konvergira.

(130) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n!}{(2n)!}$

$a_n = \frac{(-1)^n \cdot n!}{(2n)!} \qquad b_n = |a_n| = \frac{n!}{(2n)!}$

$1^\circ \quad \frac{b_n}{b_{n+1}} = \frac{\frac{\cancel{n!}}{\cancel{(2n)!}}}{\frac{(n+1)\cdot \cancel{n!}}{(2n+2)(2n+1)\cdot \cancel{(2n)!}}} = \frac{(2n+2)(2n+1)}{n+1} = \boxed{\frac{4n^2+6n+2}{n+1} >}$

$*$

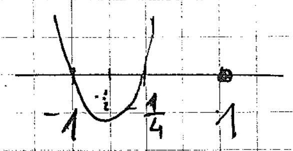
$(*) \quad \frac{4n^2+6n+2}{n+1} > 1 \quad / \; (n+1) > 0, \; \forall n$

$4n^2+6n+2 > n+1$

$4n^2+5n+1 > 0$

$n_{1,2} = \frac{-5 \pm \sqrt{25-16}}{8} \Rightarrow n_{1,2} = \frac{-5 \pm 3}{8} \quad \boxed{\begin{array}{l} n_1 = -1 \\ n_2 = -\frac{1}{4} \end{array}}$

$4n^2+5n+1 = 4(n+1)\left(n+\frac{1}{4}\right)$



$4n^2+5n+1 > 0, \; \forall n \in \mathbb{N}$

$\frac{b_n}{b_{n+1}} > 1 \Rightarrow b_n > b_{n+1} \Rightarrow$

$\Rightarrow (b_n)$ je opadajući

$2^\circ \quad \lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n!}{(2n)!} = \lim_{n\to\infty} \frac{n!}{1 2 \cdots n \cdot (n+1) \cdots (n+n)} =$

$= \lim_{n\to\infty} \frac{1}{n(n+1)\cdots(2n)} = 0$

Iz 1° i 2° $\Rightarrow$ red a_n konv. po Lajbnicovom kriterijumu.

131. $$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{2}{\sqrt{n^2+2}+n}$$

$$b_n = |a_n| = \frac{2}{\sqrt{n^2+2}+n}$$

$$n^2 < (n+1)^2 \Rightarrow n^2+2 < (n+1)^2+2$$

$$\sqrt{n^2+2}+n < \sqrt{(n+1)^2+2}+n$$

$$\Rightarrow \frac{2}{\sqrt{n^2+2}+n} > \frac{2}{\sqrt{(n+1)^2+2}+n}$$ $\Rightarrow$ red opada

$b_n > b_{n+1}, \forall n \in \mathbb{N}$

2° $\lim\limits_{n\to\infty} b_n = 0$

Iz 1° i 2° $\Rightarrow$ red $\sum a_n$ konvergira

132. $$\sum_{n=3}^{\infty} \frac{(-1)^{n-3}\cdot\sqrt{n}}{n+4}$$

$$b_n = |a_n| = \frac{\sqrt{n}}{n+4}$$

1° $f(x) = \frac{\sqrt{x}}{x+4}$; $f'(x) = \frac{4-x}{2\sqrt{x}\cdot(x+4)^2} < 0 \Leftrightarrow 4-x<0$

$\Leftrightarrow x > 4$

$f'(x) < 0, \forall x \in [5, +\infty)$ f je opadajuća na $[5, +\infty)$;

$n < n+1$

$f(n) > f(n+1)$

$b_n > b_{n+1}, \forall n \geq 5$

2° $\lim\limits_{n\to\infty} b_n = 0$

iz 1° i 2° $\Rightarrow$ red $\sum a_n$ konv

Abelov i Dirihleov krit. – nemaju veze sa alternativnim redovima

133. $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\ln^2 n} \cdot \cos\frac{\pi}{n+1}$

$a_n = \frac{(-1)^{n-1}}{\ln^2 n}$, $b_n = \cos\frac{\pi}{n+1}$

red konv. niz b_n monoton i ograničen

⇒ red $a_n \cdot b_n$ konverg.

I $\sum_{n=2}^{\infty} a_n$ je alternativni – Lajbnic

$c_n = |a_n| = \frac{1}{\ln^2 n}$

1° $\ln^2 n < \ln^2(n+1), \forall n \geq 2$

$\frac{1}{\ln^2 n} > \frac{1}{\ln^2(n+1)}, \forall n \geq 2$

$c_n > c_{n+1}, \forall n \geq 2$; c_n je opadajući

2° $\lim_{n\to\infty} c_n = 0$

iz 1° i 2° ⇒ a_n konvergira

II 1° $|b_n| = \left|\cos\frac{\pi}{n+1}\right| \leq$ ⇒ niz je ograničen

2° monoton : $n+1 < n+2$

$\frac{\pi}{n+1} > \frac{\pi}{n+2}, \forall n \geq 2$

$\frac{\pi}{n+1} \in \left(0, \frac{\pi}{2}\right), \forall n \geq 2$

Na $\left(0, \frac{\pi}{2}\right)$ $\cos x$ je opadajuća fja

$\cos\frac{\pi}{n+1} < \cos\frac{\pi}{n+2}$; $b_n < b_{n+1}, \forall n \geq 2$ ⇒

⇒ b_n je monotono rastući

I $\sum_{n=2}^{\infty} a_n$ konv II b_n monot i ogr ⇒ $\sum_{n=2}^{\infty} a_n \cdot b_n$ konverg.

134. Dirihleov kriterijum

$$\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^n \cdot \frac{1}{n}$$ pokazati da je a_n niz parcijalnih suma i da je b_n opadajući i teži 0

$a_n = \left(-\frac{1}{2}\right)^n$, $b_n = \frac{1}{n}$

I 1° $\frac{1}{n} > \frac{1}{n+1}$, $\forall n$

$b_n > b_{n+1}$, $\forall n$ $\Rightarrow$ b_n je opadajući

2° $\lim_{n\to\infty} b_n = 0$

II $\sum_{n=1}^{\infty} a_n$ → niz parcijalnih suma ograničen

$S_n = a_1 + a_2 + \dots + a_n$

$S_n = -\frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + \left(-\frac{1}{2}\right)^n$

$S_n = -\frac{1}{2} \cdot \frac{1-\left(-\frac{1}{2}\right)^n}{1-\left(-\frac{1}{2}\right)}$

$S_n = -\frac{1}{2} \cdot \frac{2}{3} \cdot \left(1-\left(-\frac{1}{2}\right)^n\right)$

$S_n = -\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^n\right)$

$|S_n| = \frac{1}{3}\underbrace{\left|1-\left(-\frac{1}{2}\right)^n\right|}_{\text{najviše } \frac{3}{2}} \le \frac{1}{3}\cdot\frac{3}{2} \le \frac{1}{2}$

$\Rightarrow$ niz (S_n) je ograničen

$$\underbrace{\text{I } 1^\circ (b_n) \text{ opad. } 2^\circ \lim_{n\to\infty} b_n = 0 \quad \text{II } (S_n) \text{ je ogr; } S_n = a_1 + \dots + a_n}$$

na osnovu Dirihleovog krit $\sum_{n=1}^{\infty} a_n b_n$ konv.

FURIJEOVI REDOVI

135. Funkciju $f(x)=\begin{cases}\pi, & -\pi\le x<0\\ x, & 0\le x\le \pi\end{cases}$

razviti u Furijeov red na $(-\pi,\pi)$

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cdot\cos\frac{2n\pi}{b-a}x+b_n\sin\frac{2n\pi}{b-a}\right)$$

$\rightarrow a_0=\frac{2}{b-a}\int_a^b f(x)dx$

$\rightarrow a_n=\frac{2}{b-a}\int_a^b f(x)\cos\frac{2n\pi}{b-a}x\,dx$

$\rightarrow b_n=\frac{2}{b-a}\int_a^b f(x)\sin\frac{2n\pi}{b-a}x\,dx$

$\rightarrow$ ni parna ni neparna

$a=-\pi$

$b=\pi$

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{\cancel{2}n\cancel{\pi}}{\cancel{2\pi}}x+b_n\cos\frac{2n\pi}{2\cancel{\pi}}x\right)$$

$$=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos nx+b_n\cos nx\right)$$

$$a_0=\frac{2}{2\pi}\int_{-\pi}^{\pi}f(x)dx=\frac{1}{\pi}\int_{-\pi}^{0}f(x)dx+\frac{1}{\pi}\int_{0}^{\pi}f(x)dx$$

$$a_0=\frac{\cancel{2}\,1}{\cancel{2}\pi}\left(\int_{-\pi}^{0}\pi\,dx+\int_{0}^{\pi}x\,dx\right)=\frac{1}{\pi}\left(\pi^2+\frac{\pi^2}{2}\right)=$$

$$=\frac{3\pi}{2}$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx=\int_{-\pi}^{0}\cos nx\,dx+\frac{1}{\pi}\int_{0}^{\pi}x\cos nx\,dx=$$

$U=x\rightarrow du=dx$

$V=\frac{1}{n}\sin nx$

$$=+\frac{1}{n}\sin nx\Big|_{-\pi}^{0}+\frac{1}{\pi}\int_0^{\pi}x\cos nx\,dx=\Big[\ U=x\rightarrow dU=dx$$

$V=\int\cos nx\,dx=$

$=\frac{1}{n}\sin nx\ \Big]$

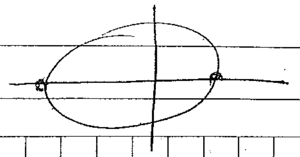
$\frac{1}{n}\sin nx\Big|_{-\pi}^{0}\rightarrow 0$

$$=\frac{1}{\pi}\cdot\frac{x\sin nx}{n}\Big|_0^{\pi}-\frac{1}{\pi}\cdot\frac{1}{n}\int_0^{\pi}\sin nx\,dx=$$

$$=\frac{1}{\pi n^2}\cos nx\Big|_0^{\pi}=\frac{1}{n^2\pi}\left(\cos n\pi-\cos 0\right)=$$

$\{\ 0,\ n$ parno

$1\ \left((-1)^n-1\right)=\{\ 0$ n neparu



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin \frac{2n\pi}{2\pi} x \, dx =$$

$dU = dx$

$V = -\frac{1}{n} \cos nx$

$$= \frac{1}{\pi} \int_{-\pi}^{0} \pi \sin nx \, dx + \frac{1}{\pi} \int_{0}^{\pi} x \sin n\cancel{\pi} x \, dx =$$

$$= \int_{-\pi}^{0} \sin nx \, dx + \frac{1}{\pi} \left(- \frac{x \cos nx}{n} \Big| + \frac{1}{n} \int_{0}^{\pi} \cos nx \, dx \right) =$$

$$= -\frac{1}{n} \cos nx \Big|_{-\pi}^{0} + \frac{1}{\pi} \left(- \frac{x \cos nx}{n} \Big|_{0}^{+\pi} \cancel{+ \frac{1}{n^2} \sin nx \Big|_{0}^{\pi}} \right) \; 0$$

$$= -\frac{1}{n} \cdot \left(\cos 0 - \cos(-\pi n) \right) + \frac{1}{\pi} \left(- \frac{\pi \cdot \cos \pi n}{n} \cancel{+ \frac{1}{n^2}} \right) =$$

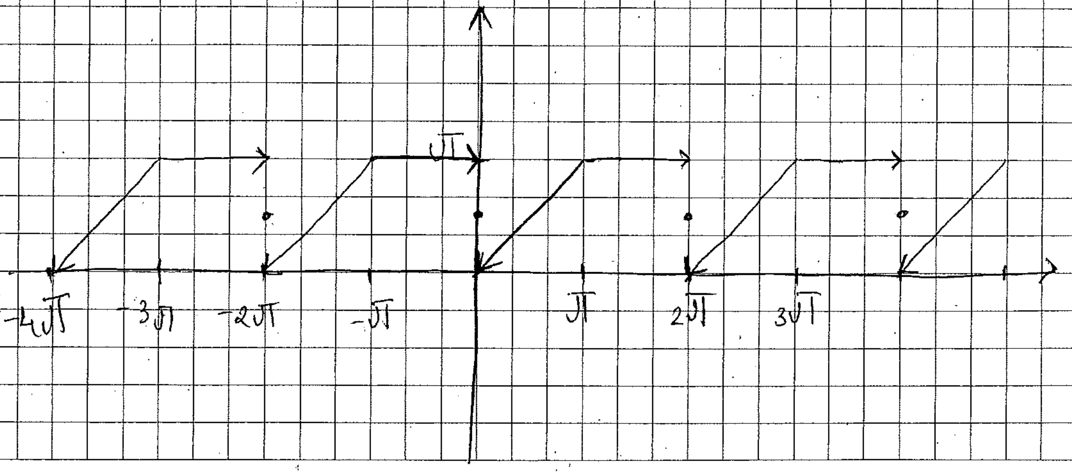
$$= -\frac{1}{n} \left(\cancel{co}\, 1 - \cos(-\pi n) \right) - \frac{\cos \pi n}{n} =$$

$$= -\frac{1}{n} + \frac{\cos \pi n}{n} - \frac{\cos \pi n}{n} = -\frac{1}{n}$$

→ Furijeov red

$$f(x) = \frac{3\pi}{4} + \sum_{n=1}^{\infty} \left(-\frac{2}{(2n-1)^2 \cdot \pi} \cos (2n-1) x \right) +$$

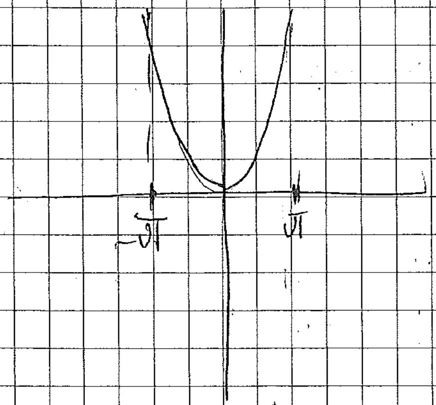
$$+ \left(\sum_{n=1}^{\infty} (-2) \cdot \frac{1}{n} \sin nx \right), \quad x \in (-\pi, \pi)$$



(136.) Razviti u Furijeov red funkciju $f(x)=5x^2$ na intervalu $(-\pi,\pi)$

$f(x)=5x^2 \longrightarrow$ funkcija je parna $\longrightarrow \forall n\in\mathbb{N},\ b_n=0$

$\longrightarrow$ ~~kao~~ razvijamo samo po cosinusima



$f(x)=5x^2,\quad x\in(-\pi,\pi)$

$$a_0=\frac{2}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx=\frac{1}{\pi}\int_{-\pi}^{\pi}5x^2\,dx=\frac{5}{\pi}\ \frac{x^3}{3}\Big|_{-\pi}^{\pi}=$$

$$=\frac{5}{\pi}\left(\frac{\pi^3}{3}+\frac{\pi^3}{3}\right)=\frac{5\cdot 2}{3\cancel{\pi}}\cdot\pi^{\cancel{3}2}=\frac{10\pi^2}{3}$$

$$a_n=\frac{2}{\pi-(-\pi)}\int_{-\pi}^{\pi} f(x)\,\cancel{dx}\,\cos\frac{2n\pi}{\pi+\pi}x\ dx=$$

$$=\frac{1}{\pi}\int_{-\pi}^{\pi}5x^2\cos nx\,dx=\begin{bmatrix}U=x^2\rightarrow dU=2x\,dx\\ V=\int\cos nx\,dx=\frac{1}{n}\sin nx\end{bmatrix}=$$

$$=\frac{5}{\pi}\left(\frac{x^2\sin nx}{n}\Big|_{-\pi}^{\pi}-\frac{2}{n}\int_{-\pi}^{\pi}x\sin nx\ dx\right)=$$

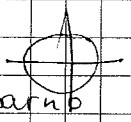
$$=\frac{5}{\pi}\left(-\frac{2}{n}\int_{-\pi}^{\pi}x\sin nx\ dx\right)=\begin{bmatrix}x=U\rightarrow dU=dx\\ V=\int\sin nx\,dx=\\ =-\frac{1}{n}\cos nx\end{bmatrix}=$$

$$= \frac{5}{\pi}\left(-\frac{2}{n}\left(-\frac{x\cos nx}{n}\Big|_{-\pi}^{\pi} + \frac{1}{n}\int_{-\pi}^{\pi}\cos nx\,dx\right)\right) =$$

$$= \frac{5}{\pi}\left(-\frac{2}{n}\left(-\left(\frac{\pi\cos n\pi}{\cancel{2}n} - \frac{-\pi\cos n\pi}{-\cancel{2}n}\right) + \frac{1}{n^2}\underset{0}{\cancel{\sin nx}}\Big|_{-\pi}^{\pi}\right)\right)$$

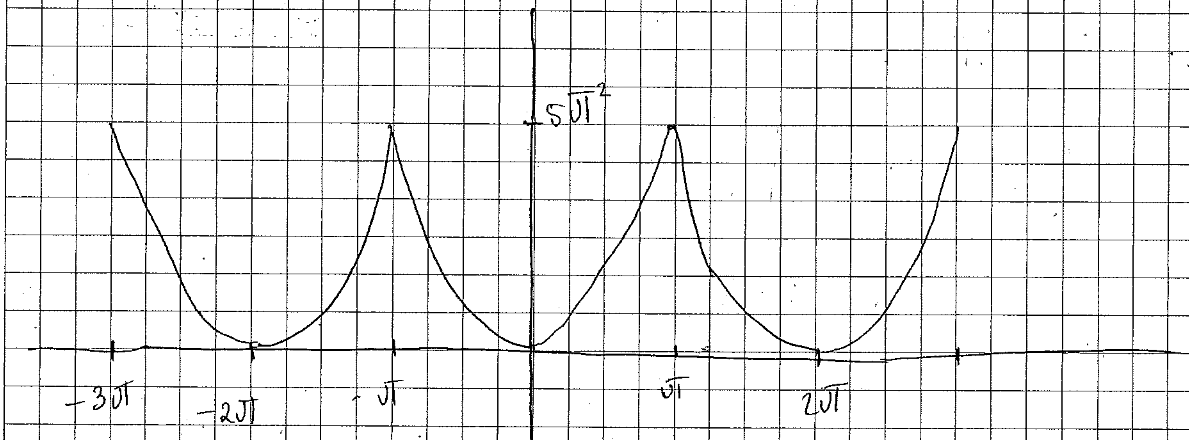
$$= \cancel{\frac{5}{\pi}\left(-\frac{2}{n}\right.} \quad \frac{5}{\pi}\left(-\frac{2}{n}\left(-\frac{2\pi\cos n\pi}{n}\right)\right) =$$

$$= \frac{20\cos n\pi}{n^2} = \frac{20}{n^2}\cdot(-1)^n = \begin{cases} \frac{20}{n^2}, & n \text{ parno} \\ -\frac{20}{n^2}, & n \text{ neparno} \end{cases}$$



$$f(x) = \underbrace{\frac{10\pi^2}{3}\cdot\frac{1}{\cancel{2}}}_{\frac{a_0}{2}} + \sum_{n=1}^{\infty} a_n \cos nx =$$

$$= \frac{5\pi^2}{\cancel{6}\,3} + 20\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cdot\cos nx \quad , \quad x \in (-\pi, \pi)$$



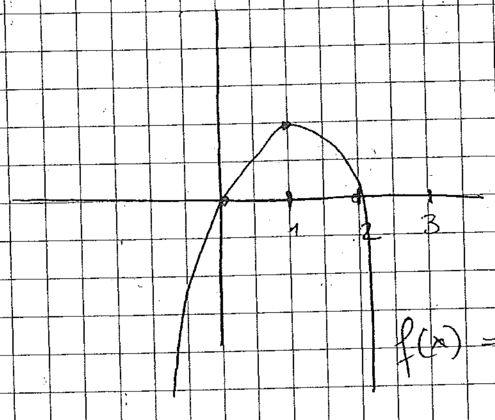
$\longrightarrow$ tacka pucanja $\rightarrow$ saberemo $\begin{smallmatrix}\nearrow 1 \\ \searrow 2\end{smallmatrix}$ $\rightarrow$ podijelimo sa 2

(137.) Funkciju $f(x) = 2x - x^2$ razviti u Furijeov red na $(0,3)$

$a = 0$
$b = 3$

$f(x) = x(2-x) \rightarrow 0, 2$

Tjeme $\rightarrow T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$



$f(x) = 2x - x^2$

$a = -1$
$b = 2$
$c = 0$

$T\left(-\frac{2}{-2}, -\frac{4}{-4}\right)$

$T(1, 1)$

$x = 3 \rightarrow y = -3$

$\rightarrow$ Razviti na intervalu $(0,3)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cdot \cos\frac{2n\pi}{3}x + b_n \sin\frac{2n\pi}{3}x\right)$$

$\rightarrow$ ~~funkcija je parna $\Rightarrow b_n = 0, \forall n \in N$~~ $\rightarrow$ nije parna!

$\downarrow$

morala bi biti parna na $(0,3)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\frac{2n\pi}{3}x$$

$$a_0 = \frac{2}{3}\int_0^3 f(x)\,dx = \frac{2}{3}\int_0^3 (2x - x^2)\,dx = \frac{2}{3}\cdot\left(x^2\Big|_0^3 - \frac{x^3}{3}\Big|_0^3\right) =$$

$$= \frac{2}{3}\cdot\left(9 - \frac{27}{9}\right) = 0$$

$$a_n = \frac{2}{3}\int_0^3 f(x)\cos\frac{2n\pi}{3}x\,dx = \frac{2}{3}\int_0^3 (2x - x^2)\cos\frac{2n\pi}{3}x\,dx =$$

$$= \frac{4}{3}\int_0^3 x\cos\frac{2n\pi}{3}x\,dx - \frac{2}{3}\int_0^3 x^2\cos\frac{2n\pi}{3}x\,dx =$$

$= ~~\frac{4}{3}~~($

$U = x \rightarrow dU = dx$

$V = \int \cos\frac{2n\pi}{3}x\,dx$

$V = \frac{3}{2n\pi}\cdot\sin\frac{2n\pi}{3}$

$U = x^2 \rightarrow dU = 2x\,dx$

$dV = \cos\frac{2n\pi}{3}x\,dx$

$V = \frac{3}{2n\pi}\sin\frac{2n\pi}{3}$

$$= \frac{4}{3}\left(\frac{3x}{2n\pi}\sin\frac{2n\pi}{3}x\Big|_0^3 - \int_0^3 \frac{3}{2n\pi}\sin\frac{2n\pi}{3}x\,dx\right) -$$

0

$$-\frac{2}{3}\left(\cancel{3}\,x^2\cdot\frac{3}{2n\pi}\sin\frac{2n\pi}{3}x\Big|_0^3 - 2\int_0^3\frac{3x}{2n\pi}\sin\frac{2n\pi}{3}x\,dx\right) =$$

0

$$= -\frac{4}{\cancel{3}}\cdot\frac{\cancel{3}}{2n\pi}\int_0^3\sin\frac{2n\pi}{3}x\,dx + \frac{4}{\cancel{3}}\cdot\frac{\cancel{3}}{2n\pi}\int_0^3 x\sin\frac{2n\pi}{3}x\,dx$$

$$= \cancel{+}\frac{4}{2n\pi}\cdot\frac{3}{2n\pi}\cos\frac{2n\pi}{3}x\Big|_0^3 \quad \frac{4}{2n\pi}\left(-\frac{x\cos\frac{2n\pi}{3}x\cdot 3}{2n\pi}\Big|_0^3 + \int\cos\right.$$

$$\left(\cos 2n\pi - \cos 0\right)$$

$1 \quad 1$

0

$2n\pi \quad 0 \quad 0$

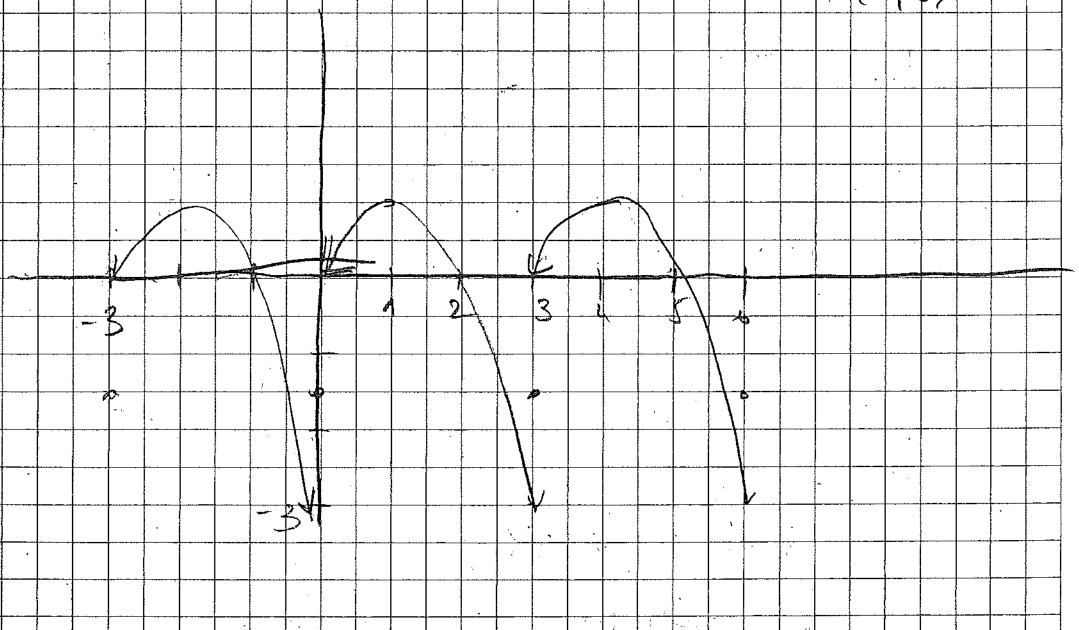
$$+\frac{4}{2n\pi}\cdot\frac{3}{2n\pi}\,\cancel{\cos}\,\sin\frac{2n\pi}{3}x\Big|_0^3 = \quad a_n = -\frac{9}{n^2\pi^2}$$

$$b_n = \frac{2}{3}\int_0^3 f(x)\sin\left(\frac{2n\pi}{3}x\right)dx = \frac{2}{3}\int_0^3(2x-x^2)\sin\left(\frac{2n\pi}{3}x\right)$$

$$b_n = \frac{3}{n\pi}$$

$$f(x) = \sum_{n=1}^{\infty}\left(\left(-\frac{9}{n^2\pi}\right)\cos\frac{2n\pi}{3}x + \frac{3}{n\pi}\sin\frac{2n\pi}{3}x\right),$$

$x \in (0,3)$



$$a_n = \frac{2}{3}\int_0^3 f(x)\cdot\cos\left(\frac{2n\pi}{3}x\right)dx$$

$$a_n = \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{2}\cdot\cancel{3}\cdot 3\cdot 3}{4n^2\pi^2}\cos 2n\pi$$

$$a_n = \frac{2}{3}\left(\underbrace{\int_0^3 (2x-x^2)\cdot\cos\left(\frac{2n\pi}{3}x\right)dx}_{I_1 - I_2}\right) = \frac{9}{n^2\pi^2}\boxed{\cancel{\cos 2n\pi}} =$$

$$\boxed{= \frac{9}{n^2\pi^2}}$$

$$\cancel{R_0}\quad I_1 = \int_0^3 2x\cdot\cos\frac{2n\pi}{3}x\,dx = \left[\begin{array}{l} U = \cancel{2}x \rightarrow du = dx \\ V = \int\cos\frac{2n\pi}{3}x\,dx = \end{array}\right.$$

$$I_1 = 2\cdot\left(\left(\frac{3x}{2n\pi}\cdot\sin\frac{2n\pi}{3}x\right)\Bigg|_0^3 - \right. \quad \left.= \frac{3}{2n\pi}\sin\frac{2n\pi}{3}x\right]$$

$$\left.\int\frac{3}{2n\pi}\sin\frac{2n\pi}{3}x\,dx\right) =$$

$$= 2\left(0 - \frac{3}{2n\pi}\int_0^3\sin\frac{2n\pi}{3}x\,dx\right) = 2\cdot\frac{3^2}{4n^2\pi^2}\cos\frac{2n\pi}{3}x\Bigg|_0^3 =$$

$$= \frac{2\cdot 9}{4n^2\pi^2}\left(\cos 2n\pi - \cos 0\right) = 0$$

$$I_2 = \int_0^3 x^2\cos\left(\frac{2n\pi}{3}x\right)dx = \left[\begin{array}{l} U = x^2 \rightarrow du = 2x\,dx \\ V = \int\cos\left(\frac{2n\pi}{3}x\right)dx = \\ V = \frac{3}{2n\pi}\sin\frac{2n\pi}{3}x \end{array}\right]$$

$$= \frac{3x^2}{2n\pi}\cdot\sin\frac{2n\pi}{3}x\Bigg|_0^3 - 2\int x\,\frac{3}{2n\pi}\sin\frac{2n\pi}{3}x\,dx =$$

$$\downarrow$$

$$= -\frac{2\cdot 3}{2n\pi}\left(-\frac{3x}{2n\pi}\cos\frac{2n\pi}{3}x\Bigg|_0^3 - \int_0^3 \frac{3}{2n\pi}\cos\frac{2n\pi}{3}x\,dx\right.$$

$$\begin{array}{l} U = x \\ dU = dx \\ V = \int\sin\frac{2n\pi}{3}x\,dx = \\ = -\frac{3}{2n\pi}\cos\frac{2n\pi}{3}x\,dx \end{array}$$

$$\boxed{= \frac{2\cdot 3\cdot 3}{4n^2\pi^2}\cos 2n\pi} - \frac{2\cdot 3\cdot 3}{4n^2\pi^2}\cdot\frac{3}{2n\pi}\cancel{\sin\frac{2n\pi}{3}x\Bigg|_0^3}\,0$$

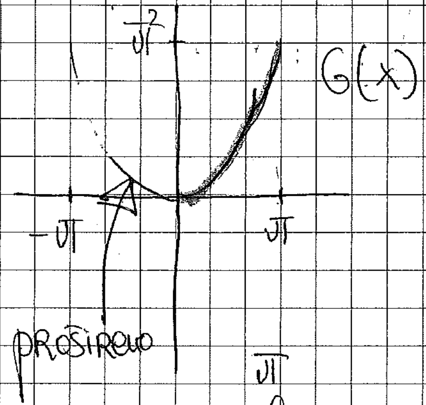
(138) Funkciju $f(x)=x^2$ razviti u kosinusni Furijeov red na $[0,\pi]$

→ f parno proširujemo

$$G(x)=\begin{cases} f(x), & x\in[0,\pi] \\ f(-x), & x\in[-\pi,0) \end{cases} =$$

$$= \begin{cases} x^2, & x\in[0,\pi] \\ (-x)^2, & x\in[-\pi,0) \end{cases} = \left\{ x^2,\ x\in[-\pi,\pi] \right.$$

→ $G(x)$ razvijamo u F. red na $[-\pi,\pi]$.



$G \to$ parna $\to$ nju razvijamo

$\underline{\underline{b_n=0}}$

$$a_0 = \frac{1}{\pi}\cdot 2\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} x^2\,dx = \frac{2}{\pi}\cdot\frac{x^3}{3}\Big|_0^{\pi} = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos\frac{2n\pi}{2\pi}x\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{x^2}_{\text{parna}}\underbrace{\cos nx}_{\text{parna}}\,dx =$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2\cos nx\,dx = \frac{2}{\pi}\left(\boxed{\frac{x^2\sin nx}{n}\Big|_0^{\pi}} - \frac{2}{n}\int_0^{\pi} \overset{\text{parna}}{x\sin nx}\,dx \right) =$$

$x^2=U \to dU=2x\,dx$

$V=\frac{1}{n}\sin nx$

$\# \ 0$

$$= -\frac{4}{n\pi}\int_0^\pi x\sin nx\,dx = \left|\begin{array}{l} x=u \to du=dx \\ v=\int \sin nx\,dx = -\frac{1}{n}\cos nx\end{array}\right| =$$

$$= -\frac{4}{n\pi}\left(-\frac{x\cos nx}{n}\Big|_0^\pi + \frac{1}{n}\int \cos nx\,dx\right) =$$

$$= -\frac{4}{n\pi}\left(-\frac{\pi\cos n\pi}{n} + \frac{1}{n^2}\underset{0}{\cancel{\sin nx}}\Big|_0^\pi\right) =$$

$$= \frac{4\cancel{\pi}}{n^2\cancel{\pi}}\cos n\pi = \frac{4}{n^2}(-1)^n$$

$$G(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}(-1)^n\cdot\cos n\pi\right), \quad x\in[-\pi,\pi]$$

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx, \quad x\in[0,\pi]$$

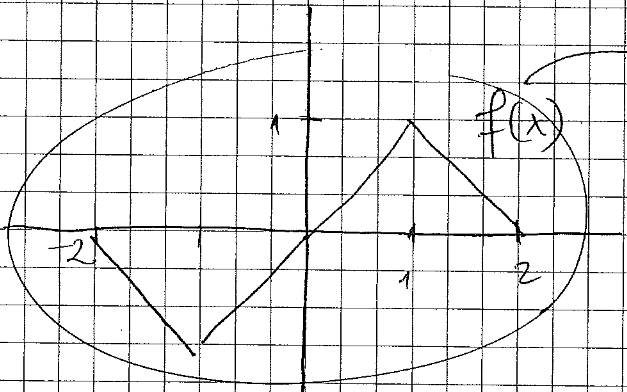
(139.) Funkciju $f(x) = \begin{cases} x, & x \in [0,1) \\ 2-x, & x \in [1,2) \end{cases}$ razvij u sinusni Furjeov red

$1-4=-3$

→ neparno proširujemo funkciju

$$F(x) = \begin{cases} f(x), & x \in [0,2) \\ -f(-x), & x \in [-2,0) \end{cases} =$$

$$= \begin{cases} x, & x \in (0,1) \\ 2-x, & x \in (1,2) \\ -(-x)=x, & x \in (-1,0) \\ -(2-(-x)) = -2-x, & x \in [-2,-1] \end{cases}$$



→ F(x) → neparno
smo je
proširili

→ F razvijamo u Furjeov red na $[-2,2]$

$a_n = 0, \ \forall n \in \mathbb{N}$ → jer je f neparna

$$a_0 = \frac{2}{4}\int_{-2}^{2} f(x)\,dx = \frac{1}{2}\int_{-2}^{-1} f(x)\,dx + \frac{1}{2}\int_{-1}^{0} f(x)\,dx + \frac{1}{2}\int_{0}^{1} f(x)\,dx$$

$$+ \frac{1}{2}\int_{1}^{2} f(x)\,dx$$

$$a_0 = \frac{2}{4}\int_{-2}^{2} F(x)$$

$$a_0 = \frac{1}{2}\int_{-2}^{-1}(-2-x)\,dx + \frac{1}{2}\int_{-1}^{0} x\,dx + \frac{1}{2}\int_{0}^{1}(2-x)\,dx + \frac{1}{2}\int_{1}^{2} x\,dx =$$

$$= -\frac{1}{2}\int_{-2}^{-1}(2+x)\,dx + \frac{1}{2}\frac{x^2}{2}\Big|_{-1}^{0} + \frac{1}{2}\cdot 2x\Big|_{0}^{1} - \frac{1}{2}\frac{x^2}{2}\Big|_{0}^{1} + \frac{1}{2}\frac{x^2}{2}\Big|_{1}^{2}$$

$$= -\cancel{\frac{1}{2}}\cancel{2}x\Big|_{-2}^{-1} - \frac{1}{2}\frac{x^2}{2}\Big|_{-2}^{-1} + \frac{1}{4}x^2\Big|_{-1}^{0} + x\Big|_{0}^{1} - \frac{1}{4}x^2\Big|_{0}^{1} + \frac{1}{4}x^2\Big|_{1}^{2} =$$

$$= -(-1+2) - \frac{1}{4}\left((-1)^2-(-2)^2\right) + \frac{1}{4}(0-1) + 1 - \frac{1}{4} + \frac{1}{4}(4-1) =$$

$$= -\cancel{1} + \frac{3}{4} - \cancel{\frac{1}{4}} + \cancel{1} - \cancel{\frac{1}{4}} + \frac{3}{4}$$

$$\rightarrow b_n = \frac{2}{4}\int_{-2}^{2}\underbrace{\underbrace{F(x)\sin\frac{n\pi}{2}x}_{\text{neparna}\cdot\text{neparna}}}_{\text{parna}}\,dx = \frac{1}{2}\cdot 2\int_{0}^{2}F(x)\sin\frac{n\pi}{2}x\,dx =$$

$$= \int_{0}^{1}x\sin\frac{n\pi}{2}x\,dx + \int_{1}^{2}(2-x)\sin\frac{n\pi}{2}x\,dx =$$

$$= \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2} = \begin{cases} 0, & n \text{ parno} \\ (-1)^k\cdot\frac{8}{(2k+1)^2\pi^2}, & n=2k+1 \end{cases}$$

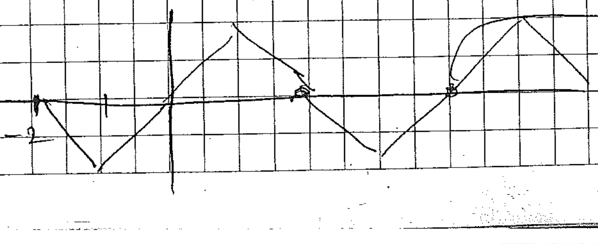
→ mora ovako zbog nule da se piše!

$$F(x) = \frac{8}{\pi^2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^2}\sin\frac{(2k+1)\pi}{2}x$$

→ $x \in [-2,2]$

$$\downarrow$$

$$f(x) = \qquad) \qquad x \in [0,2]$$



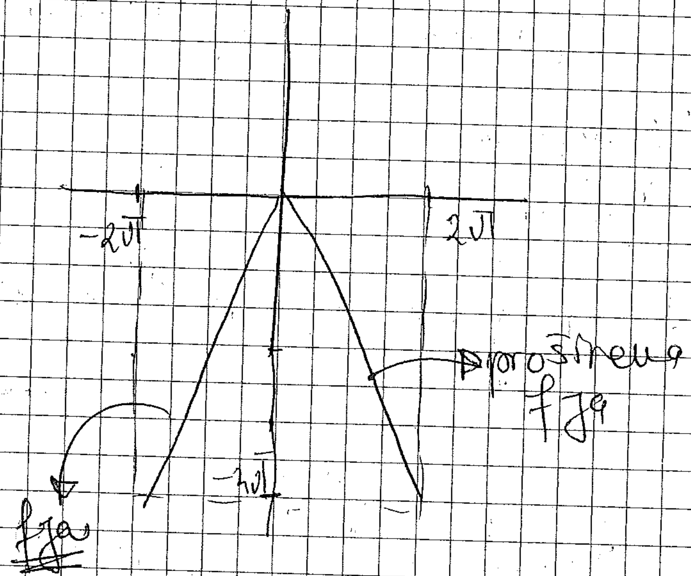
→ granične tačke

140. Razviti u red po kosinusima funkciju $f(x)=2x$ na $[-2\pi, 0)$

→ po kosinusima → f proširujemo parno

$$G(x) = \begin{cases} f(x), & x \in [-2\pi, 0) \\ f(-x), & x \in (0, 2\pi] \end{cases}$$

$$G(x) = \begin{cases} 2x, & x \in [-2\pi, 0] \\ -2x, & x \in (0, 2\pi] \end{cases}$$



G razviti na $[-2\pi, 2\pi]$

(141.) Funkciju $f(x) = -\left|\frac{3\pi}{8} - \frac{x}{2}\right|$ razvij u red po sinusima na $[0,\pi]$

$\rightarrow$ po sinusima $\rightarrow$ f neparno proširimo

$$F(x) = \begin{cases} f(x), & x \in [0,\pi] \\ -f(-x), & x \in [-\pi, 0) \end{cases}$$

$$F(x) = \begin{cases} -\left|\frac{3\pi}{8} - \frac{x}{2}\right|, & x \in [0,\pi] \\ +\left|\frac{3\pi}{8} + \frac{x}{2}\right|, & x \in [-\pi, 0) \end{cases}$$

$$-\left|\frac{3\pi}{8} - \frac{x}{2}\right| = \begin{cases} \frac{x}{2} - \frac{3\pi}{8}, & x \leq \frac{3\pi}{4} \\ \frac{3\pi}{8} - \frac{x}{2}, & x > \frac{3\pi}{4} \end{cases}$$

$$\left|\frac{3\pi}{8} + \frac{x}{2}\right| = \begin{cases} \frac{x}{2} + \frac{3\pi}{8}, & x \geq -\frac{3\pi}{4} \\ -\frac{x}{2} - \frac{3\pi}{8}, & x < -\frac{3\pi}{4} \end{cases}$$

$\rightarrow$ F se razvija na $\underset{a}{-\pi}, \underset{b}{\pi}$

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